# Exercise 1

Let us first write the predicate *P*:

Let us compute the base case:

We hence have that which is true.

Our inductive hypothesis is as follows:

We assume to prove , that is:

Since and , our inductive case will be as follows:

That is equivalent to the following:

If we divide by on both sides of the inequality, we have only 3 left on the left side. On the right side we know that since our inductive hypothesis tells us that . From the definition of *n* we know that *k* will be at least and hence we have that which is true and we have thus proven that if then .

# Exercise 2

We recall the definition of :

Our predicate is as follows:

Let us compute the base case:

Hence we have that the base case is true.

Our inductive hypothesis is that , i.e.:

For our inductive case we will assume that to prove :

We will use the definition for the Fibonacci sequence to rewrite the right side of the equation while the left side of the equation we will take out of the summation:

On the right side of the equation, we multiply the into the parenthesis:

From our inductive hypothesis, we have that and hence we can reduce to the following:

And thus we have proved our inductive case .

# Exercise 3

Our division theorem states that where .

In other words, we want to prove that for any natural number *a*, we can divide it by a positive integer *b* such that we get a quotient and a remainder where the remainder is strictly less than b.

Our predicate is as follows: .

I will divide the proof into two cases:

First, the case where :

Our base case is as follows:

Because if such that where , then and thus not in this case. That is because is only defined for any integer that is at least 0 and hence the right side of the equation will at least be or greater.

In other words, if and only if a is strictly less than b then the remainder will always be equal to a and the quotient will be 0.

In our second case :

We will consider the above as our base cases, i.e. .

Inductive Hypothesis: To prove , we assume .

In particular, we assume is true. That is possible because is a natural number if and only if and and thus following our inductive hypothesis.

Then to prove .

We will then add b to either side of our equation sign:

We will factorize b:

We know that and thus . Then let and we have:

We have thus proven that any natural number dividend can be divided by any positive integer divisor such that it produces a quotient and a remainder .